Section 12.10

Solving Probability Problems by Using Combinations

Example

- A club consists of 5 men and 6 women. Four members are to be selected at random to form a committee. What is the probability that the committee will consist of two women?

$$P\begin{pmatrix}\text{two}\\ \text{woman}\end{pmatrix} = \frac{\text{\# of possible committees with 2 woman}}{\text{total number of 4-member committees}}$$

$$= \frac{{}_6C_2}{{}_{11}C_4} = \frac{\frac{6!}{4!2!}}{\frac{11!}{7!4!}} = \frac{\cancel{6}^{3}\cdot 5\cdot\cancel{4\cdot 3\cdot 2\cdot 1}}{\cancel{4\cdot 3\cdot 2\cdot 1}\cdot\cancel{2\cdot 1}}\cdot\frac{\cancel{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}\cdot 4\cdot 3\cdot 2\cdot 1}{11\cdot 10\cdot\cancel{9}_{3}\cdot\cancel{8}\cdot\cancel{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}}$$

$$= \frac{15}{330} = \frac{1}{22}$$

Example

- A bag contains four red balls and five green balls. You plan on selecting three balls at random. Determine the probability of selecting three green balls.

$$P\ (3\ green\ balls) = \frac{{}_5C_3}{{}_9C_3}$$
$$= \frac{10}{84} = \frac{5}{42}$$

Example

- You are dealt 5 cards from a standard deck of 52 cards. Determine the probability that you are dealt 5 red cards.

$$P\ (5\ red\ cards) = \frac{{}_{26}C_5}{{}_{52}C_5}$$
$$= \frac{65780}{2598960}$$
$$= \frac{253}{9969}$$

Example

- The Honey Bear is testing 10 new flavors of ice cream. They are testing 5 vanilla based, 3 chocolate based and 2 strawberry based ice creams. If we assume that each of the 10 flavors has the same chance of being selected and that 4 new flavors will be produced, find the probability that

  a) no chocolate flavors are selected.

  b) at least 1 chocolate is selected.

  c) 2 vanilla and 2 chocolate are selected.

Solution

- 5 vanilla, 3 chocolate, 2 strawberry

  selecting 4 flavors

a) $P(\text{no chocolate}) = \dfrac{{}_7C_4}{{}_{10}C_4} = \dfrac{35}{210} = \dfrac{1}{6}$

b) $P(\text{at least 1 chocolate}) = 1 - P(\text{no chocolate})$

$$= 1 - \frac{1}{6} = \frac{5}{6}$$

Solution (continued)

- 5 vanilla, 3 chocolate, 2 strawberry; selecting 4 flavors

c. $$P\begin{pmatrix}\text{2 vanilla and}\\ \text{2 chocolate}\end{pmatrix} = \frac{{}_5C_2 \cdot {}_3C_2}{{}_{10}C_4}$$

$$= \frac{10 \cdot 3}{210} = \frac{30}{210}$$

$$= \frac{1}{7}$$

Example

- An airline is given permission to fly 4 new routes of its choice. The airline is considering 12 new routes: 4 routes in FL,5 routes in CA, and 3 routes in TX. If the airline selects the 4 new routes at random from the 12 possibilities, determine the probability that
  - 2 are in FL and 2 are in TX
  - 3 are in CA and 1 is in FL
  - 1 is in FL, 1 is in CA, and 2 are in TX
  - At least one is in TX

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  - 2 are in FL and 2 are in TX $\frac{2}{55}$
  - 3 are in CA and 1 is in FL
  - 1 is in FL, 1 is in CA, and 2 are in TX
  - At least one is in TX

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  - 2 are in FL and 2 are in TX $\frac{2}{55}$
  - 3 are in CA and 1 is in FL $\frac{8}{99}$
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  - At least one is in TX

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  - 2 are in FL and 2 are in TX $\frac{2}{55}$
  - 3 are in CA and 1 is in FL $\frac{8}{99}$
  - 1 is in FL, 1 is in CA, and 2 are in TX $\frac{4}{33}$
  - At least one is in TX

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  - 3 are in CA and 1 is in FL $\frac{8}{99}$
  - 1 is in FL, 1 is in CA, and 2 are in TX $\frac{4}{33}$
  - At least one is in TX $\frac{41}{55}$